DARCY MULTI-DOMAIN APPROACH FOR INTEGRATED SURFACE/SUBSURFACE HYDROLOGIC MODELS : APPLICATION TO CATCHMENT HYDROLOGY AND IMPACT OF CLIMATE CHANGE

APPROCHE MULTI DOMAINE POUR LES ECHANGES SURFACE SUBSURFACE APPLICATION A L'HYDROLOGIE DE BASSINS VERSANTS ET A L'IMPACT DU CHANGEMENT CLIMATIQUE

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Abstract: The main processes governing hydrological cycle at the catchment scale may be divided in two categories: surface processes and subsurface processes. Surface ones, such as runoff are strongly coupled to subsurface ones, like infiltration in the vadose zone or seepage. Moreover water dynamics between surface and subsurface is very sensitive to rainfall, and therefore to annual climatic variability. Therefore, coupled simulations of surface and subsurface flows are necessary for an accurate modelling of real hydrological situations. Until recently, hydrologic models assumed a weak coupling, or no coupling at all, between surface and subsurface, essentially for numerical reasons: (i) runoff kinetics is much faster than infiltration and groundwater kinetics; (ii) for subsurface modelling runoff dynamics provides the variable boundary condition, both in time and space, at the soil surface. Since a few years, hydrologic modellers are developing models that take into account all processes together.
We present in this paper such a model. It is based on a Darcy multi-domain approach in which the diffusive wave approximation is used to model runoff. The resulting equation is formulated as a Darcy nonlinear one. Therefore, the water dynamics in the three physical domains, ground surface, vadose zone and saturated zone, is described through a single Darcy nonlinear equation with domain-dependent parameters. We show how this equation is solved numerically and applied to different 2D and 3D catchment hydrologic situations. This physical and mathematical formulation may provide an interesting framework for the study of the impact of rainfall annual variability on the water budget in a catchment.

Keywords : Hydrological model, catchment, surface subsurface dynamics

Résumé : Les principaux processus hydrologiques dans un bassin versant peuvent être classés en deux catégories : les processus de surface et ceux de subsurface. Les processus de surface, tels que le ruissellement, sont fortement couplés à ceux de surface tels que l'infiltration ou le suintement. De plus l'échange d'eau entre surface et subsurface est très sensible à la pluie, et, dès lors, à la variabilité pluviométrique annuelle. Par conséquent, les simulations couplant surface et subsurface sont nécessaires pour modéliser de façon réaliste le cycle hydrologique. Jusqu'à récemment les modèles hydrologiques ne prenaient pas en compte, ou de façon faible, le couplage entre surface et subsurface, essentiellement pour des raisons numériques : (i) la cinétique des phénomènes de surface est plus rapide

que celle des phénomènes de subsurface ; (ii) les phénomènes de surface fournissent les conditions aux limites en espace et en temps pour les phénomènes de subsurface. Depuis peu des modèles couplant tous les phénomènes apparaissent dans la littérature.
Nous présentons dans ce papier un tel modèle. Il est basé sur une approche darcéenne multi domaine dans laquelle le ruissellement est traité dans le cadre de l'onde diffusive. L'équation correspondante est traitée comme une équation de Darcy non linéaire. Dès lors la dynamique de l'eau dans les trois compartiments, surface, zone non saturée et zone saturée est décrite par une seule équation de Darcy non linéaire avec des lois et paramètres multi domaines. Nous montrons comment cette équation est résolue numériquement et nous l'appliquons à des situations hydrologiques 2D et 3D. Ce modèle fournit un cadre permettant d'étudier l'impact de la variabilité pluviométrique annuelle sur le cycle de l'eau dans un bassin versant.

Mots clés : Modèle hydrologique, bassin versant, échanges surface subsurface

INTRODUCTION

The main processes governing the hydrological cycle at the catchment scale may be divided in two categories: surface processes and subsurface processes. Surface ones, such as runoff, are strongly coupled to subsurface ones, such as infiltration in the vadose zone or seepage. Therefore, coupled simulations of surface and subsurface flows are necessary for an accurate modelling of real hydrological situations. From a fluid mechanics point of view, these processes can be described as follow: when rainfall occurs, rainfall water can either infiltrate or participate in surface runoff, depending on the soil moisture. If rainfall intensity exceeds what Horton calls the soil infiltration capacity (Horton 1933), water flows on top of the soil surface and eventually reaches the streamflow. Runoff water can sometimes re-infiltrate where the soil is unsaturated along its flow path. If rainfall intensity is smaller than the infiltration capacity, water infiltrates into the vadose zone to the aquifer and then flows down to the stream through the saturated zone. If the water table is just below the soil surface, the increase of the groundwater level due to infiltration can lead to seepage. In this case, water that has first infiltrated, participates afterwards to surface runoff. Water can also flow at the soil surface if the rain falls on a saturated area or if exfiltration occurs (Kirkby, 1978).
Until recently, hydrologic models did not take into account the coupling between surface and subsurface, essentially for two numerical reasons: (i) runoff kinetics is much faster than the infiltration and groundwater flow kinetics; (ii) for subsurface modelling, runoff dynamics provides the variable boundary condition, both in space and time, at the soil surface (Beaugendre et al, 2006). Since a few years, hydrologic modellers are developping models that take into account those processes altogether (Vanderkwaak and Loague 2001, Panday and Huyakorn 2004, Beaugendre et al, 2006). The main difference between these models is the way the surface/subsurface (S/SS) coupling is implemented. In the past, some models proposed at best a weak coupling: depending on the soil properties, a rainfall fraction infiltrated and the other part flowed to the stream by means of a routing procedure (Beven 2004). Today, some use a first-order law to quantify the exchange terms (Vanderkwaak, 2001, Panday, 2004) others prescribe pressure and velocity continuity at the models interface (Beaugendre et al, 2006). The development of

such models is a difficult task because water dynamics is highly nonlinear and the S/SS kinetics are very different. Moreover, the hydrological response is highly dependent on the type of soil. One should mention that the validation of such models strongly suffers from a lack of experimental results and analytical solutions. Nevertheless, two test cases stand today as references in surface/subsurface modelling: an experimental one (artificial rain falling on a sandbox) described in Abdul and Gillham (1984) and a 2D theoretical hillslope system described in Ogden and Watts (2000).

Following the above cited authors, we propose a new model that can take into account all interactions between surface and subsurface (Weill and Mouche 2006). We assume that runoff occurs in a layer at the top of the soil surface. Overland flow is modelled using the diffusive wave approximation, which is considered as a nonlinear Darcy equation. Thus, all surface and subsurface processes are described by a single nonlinear Darcy equation with domain-dependent parameters. In this way, interactions can be treated in a continuous way: pressure and velocity are continuous through the soil surface. Thus, there is no need to introduce a first order S/SS coupling law.

PHYSICAL MODEL

Overland flow

For sake of simplicity, the model presented below is written for a 2D hillslope vertical cross-section. The shallow water approximation leads to the mass balance equation (Kirkby, 1978):

$$\frac{\partial h_r}{\partial t} + \frac{\partial q}{\partial t} = 0 \qquad (1)$$

where x is the axis coordinate along the slope, h_r is the water depth and $q = h_r V$ the runoff discharge in which V is the flow velocity. This one is determined with the Manning-Strickler uniform flow formula. Within the diffusive wave approximation V is written (Kirkby 1978):

$$V = \frac{h_r^{2/3}}{n\sqrt{S}} \frac{\partial (z_g + h_r)}{\partial x} \qquad (2)$$

where n is the Manning-Strickler coefficient, S the soil slope and z_g the ground elevation. Combining equation (1) and (2), we obtain the diffusive wave equation:

$$\frac{\partial h_r}{\partial t} + \frac{\partial}{\partial x}\left(\frac{h_r^{5/3}}{n\sqrt{S}} \frac{\partial (z_g + h_r)}{\partial x} \right) = 0 \qquad (3)$$

This equation looks like a generalized Darcy's law with a nonlinear hydraulic conductivity equal to $K_r = h_r^{5/3} / n\sqrt{S}$

Unsaturated and saturated flows

Flows in unsaturated and saturated zones are described by Darcy's law, written as:

$$C(h_s)\frac{\partial H}{\partial t} + \vec{\nabla}.(K(h_s)\overline{\nabla} H) = 0 \qquad (4)$$

where $H=h_s+z$ is the total hydraulic head, h_s is the capillary pressure, z is the elevation, $C(h_s)$ is the soil capillary capacity and $K(h_s)$ is the hydraulic conductivity. In the saturated zone, $K(h_s)$ is constant and in the vadose zone, depends on capillary pressure.

NUMERICAL RESOLUTION

The simulation domain is divided in two sub-domains: the runoff layer and the soil, where equations (3) and (4) apply respectively. In both domains, variables have the same meaning and therefore are continuous at the runoff layer/soil interface. The set of flow equations is solved with respect to the total hydraulic head H.

The runoff mass balance equation (3) has to be transformed to account for all S/SS processes and for this equation to be valid for all h values. In the runoff layer, water depth h_r is defined using the total hydraulic head H and the soil elevation z_g : $h_r = H - z_g$. A negative value of h_r means that there is no water in the runoff layer and, consequently, no runoff. In this case, the layer hydraulic conductivity is supposed to be equal to zero. However, we cannot define a hydraulic conductivity equal to zero in our numerical formulation. So, we set the permeability to a residual value ε as small as possible for our model to converge. A positive value of h_r means that runoff takes place in the layer. In this case, the horizontal hydraulic conductivity in the runoff layer is equal to $K_r(h_r) + \varepsilon$.
In the vertical direction, i.e. in the infiltration direction, we impose a high vertical permeability K_{zz} so that water flows "instantaneously" through the runoff layer into the soil. If rainfall intensity is smaller than the soil infiltration capacity, i.e. in a non Hortonian regime, water flux at the runoff layer/soil interface is equal to the one imposed at the top of the runoff layer. If rainfall intensity is higher than the infiltration capacity, i.e. in a Hortonian regime, flux at the runoff layer/soil interface is equal to the one imposed at the top of the runoff layer until the top of the soil becomes fully saturated. When that happens, a fraction of rainfall water infiltrates due to K_{zz}, while the other fraction stays in the runoff layer and then participates in overland flow. Considering these transformations, runoff equation (3) can be rewritten:

$$C(h_r)\frac{\partial H}{\partial t}+\frac{\partial}{\partial x}\left(\left(\frac{h_r^{5/3}}{n\sqrt{S}}+\varepsilon\right)\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial H}{\partial z}\right)=0 \quad (5)$$

where $C_r(h_r)$ equals 1 when h_r is positive and 0 when h_r is negative.
Combining equations (6) and (7), we obtain for the whole domain, including the runoff layer and the soil domain, a single nonlinear Darcy equation written:

$$\tilde{C}(\tilde{h})\frac{\partial H}{\partial t}+\frac{\partial}{\partial x}\left(\tilde{K}_x(\tilde{h})\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial z}\left(\tilde{K}_z\frac{\partial H}{\partial z}\right)=0 \quad (6)$$

This equation is solved using a Mixed Hybrid Finite Element formulation. The time discretization is implicit and the nonlinear terms are solved within an iterative Picard algorithm (Lepotier, 1998):

$$C(h^{n+1,i})\frac{H^{n+1,i+1}-H^{n}}{\Delta t}=-\vec{\nabla}.(K(h^{n+1,i})\vec{\nabla}H^{n+1,i+1} \quad (7)$$

where n is the time step index and i the iteration index.. The model and this algorithm has been implemented in the finite element code CAST3M (www-cast3m.cea.fr), which is a general computational tool developed at the CEA for mechanics and fluid mechanics applications.

VALIDATION-APPLICATION

We present in this paper two applications of our model, which can be considered as validation test cases too. These are the Abdul and Gillham's system (Abdul and Gillham 1984) and the Ogden and Watts system (Ogden and Watts 2000).

Abdul and Gillam's system

The system described in Abdul and Gillham (1984) is a reference system for modellers interested in S/SS interactions. We use it to validate our model. This experimental system was designed to study the influence of capillary fringe extension on the runoff generation processes. The system is a simple sandbox with no flow boundaries except at the soil surface (Fig. 1). Soil surface is thus the only interface that allows water to enter or exit the system, that is what makes this system quite hard to model. The sand box is 1.4 meter wide and from 0.8 to 1 meter high, the slope of the surface is around 14%. The material is Yolo Light Clay. We assume a Manning coefficient value of 0.1. The imposed flux at the top of the domain is 10 % of the saturated permeability and the groundwater table is initially at the toe of the slope.

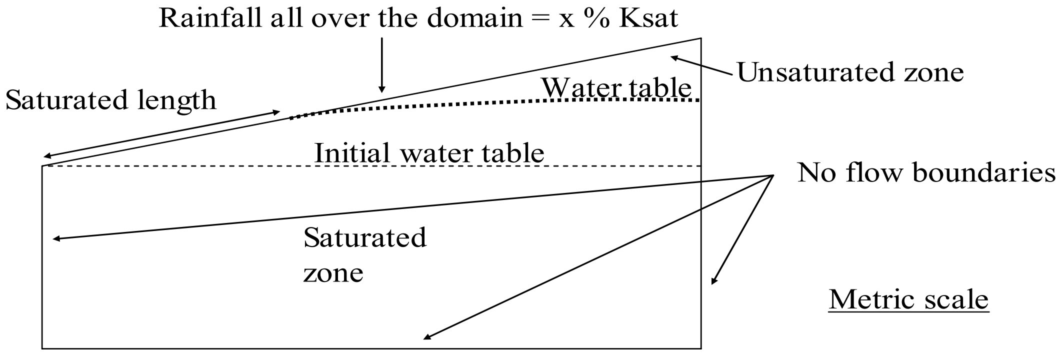


Fig. 1. Abdul and Gillham system.

The observed dynamics of the system is the following: initially, the groundwater table is at the toe of the slope, but, due to capillarity, the soil is nearly saturated in the whole domain. As a consequence, the rainfall water first infiltrates but a small amount of infiltrated water is enough to make the groundwater level exceed the soil surface elevation at the toe of the slope. The infiltrated water at the top of the slope induces a head gradient that produces exfiltration. Rain which falls on saturated areas is not able to infiltrate anymore and then flows to the outlet in the runoff layer. Then, the infiltrated flux decreases as both the exfiltration flux and the saturated length begin to increase. At the steady state, the saturated length has reached a constant value and the exfiltrated and infiltrated fluxes are equal. We also observe that at early times the infiltrated flux is not equal to 1. This is due to the ε residual hydraulic conductivity imposed in the negative water depth areas of the runoff layer. If we want our system to converge, the permeability contrast between dry and wet areas of the runoff layer must not be too large, namely three or four orders of magnitude. Therefore, a small percentage of rainfall flows in the layer instead of infiltrating. Figures 2(a) shows the water pressure distribution and 2(b) the normalized Darcy velocity distribution at steady state. We observe the three different surface regimes observed by Abdul and Gillham in their experiments. At the top of the slope, the velocity is vertical and downwards, which means that water is only infiltrating. At the toe of the slope, velocity is vertical and upwards, showing that water is exfiltrating. In the middle of the slope, the velocity vectors are neither vertical, nor horizontal, which means that a fraction of rainfall is infiltrating and the other is flowing on the surface. This velocity field is in good agreement with the one presented in Abdul and Gillham [1984]. Figure 3(a) displays the time evolution of the normalized infiltrated and exfiltrated fluxes through the soil surface and 3(b) the time evolution of the normalized saturated length along the slope for three classical soils (Beaugendre et al, 2006). Fluxes are normalized by the rain intensity and the saturated length by the slope length

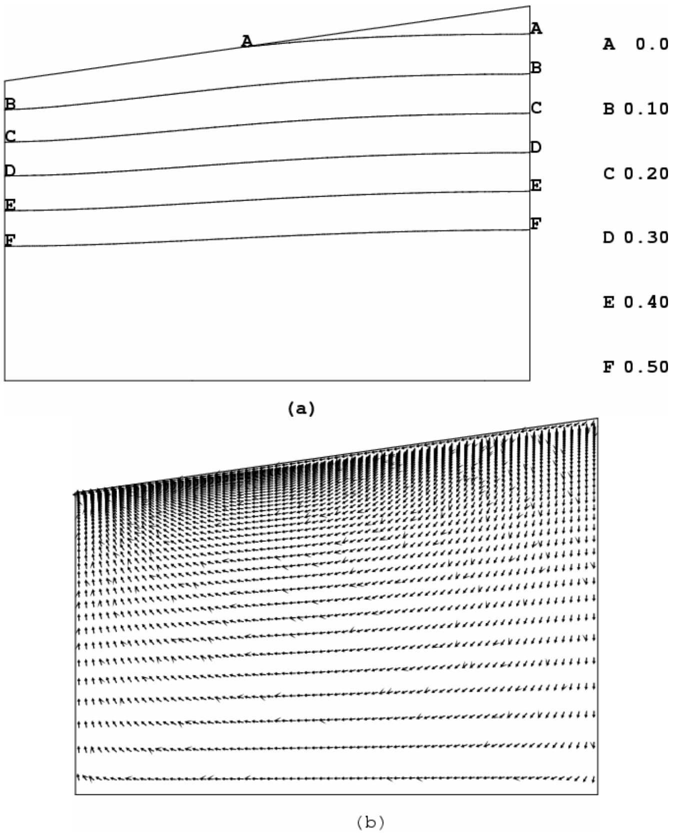
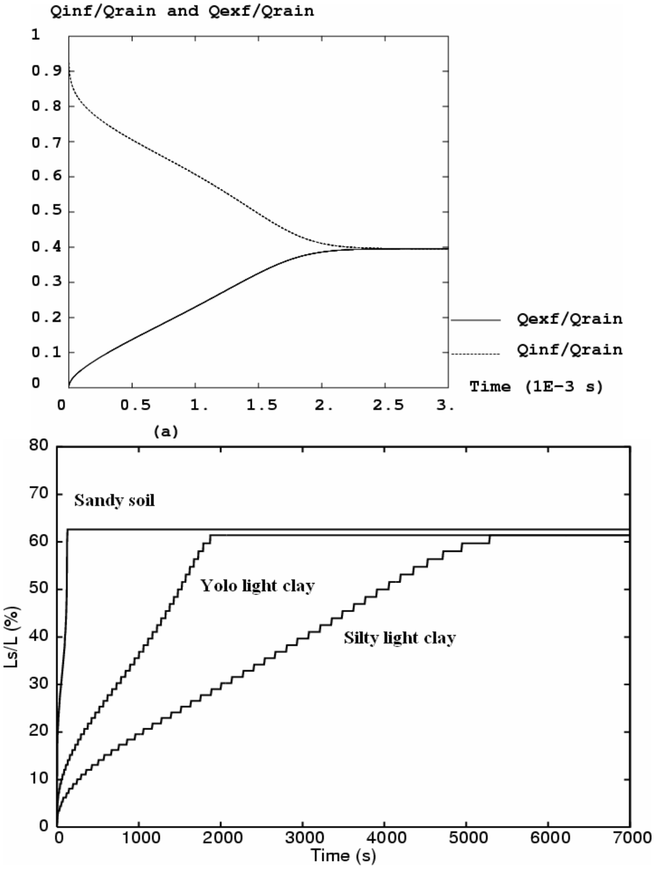


Fig. 2. (a) Water pressure in m. and (b) Normalized velocity distribution.



(b)

Fig. 3. (a) Time evolution of the normalized flux (dot line) and exfiltrated flux (solid line) and (b) time evolution of the normalized saturated length for three soils.

Ogden and Watts system

The second system studied here has been proposed by Ogden and Watts (Ogden and Watts 2000). It is a hypothetical modelling system designed to study the formation of saturated area in watersheds with shallow highly pervious soils. It is a sloping vertically oriented two dimensional system, as shown in figure 4. A no flow boundary condition is employed at the upslope end, representing a watershed divide. A constant head boundary condition is used at the downslope end, representing a stream. The upper part of the domain allows infiltration and exfiltration. In their paper Ogden and Watts investigated numerically the hydrologic response of the system for a given rainfall and its dependence with respect to soil parameters and system dimensions, namely L and D (see figure 4). The two principal outputs of their modelling were the transit time to reach the steady state and the time evolution of the saturated length. Figure 5 shows the dependency of the normalized saturated length with respect to parameters D and L. We see that shallower is the soil greater is the value of the length, which is quite easy to understand. On figure 6 we show the hydrograph for a rainfall event (D=1m and L=30m) and the surface and subsurface contributions. We see as, expected, that runoff gives a rapid and important contribution to the hydrograph peak and the subsurface compartment drives the long term tail of the hydrograph.

3D simulations have been too performed. One of the advantage of this modelling approach is that streams are surface hydrologic components : a stream is treated physically and numerically like surface runoff with proper parameters. Therefore a stream is incorporated in the runoff layer and treated the same way as runoff.

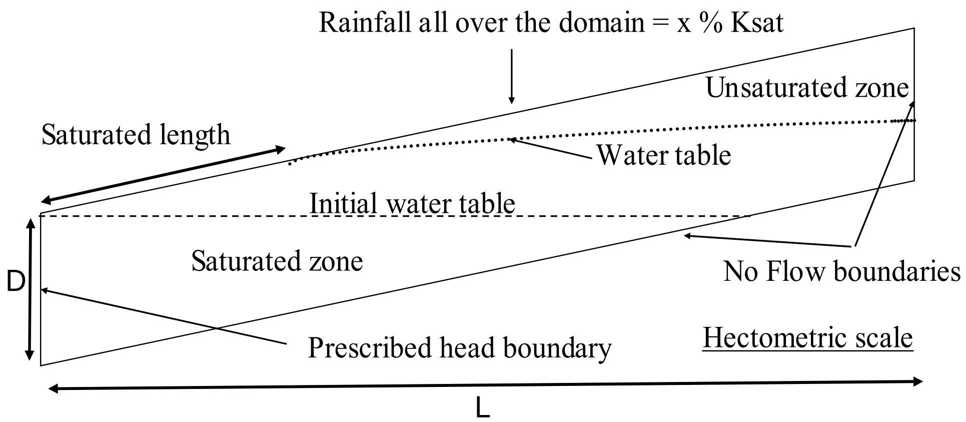


Fig. 4. Ogden and Watts system

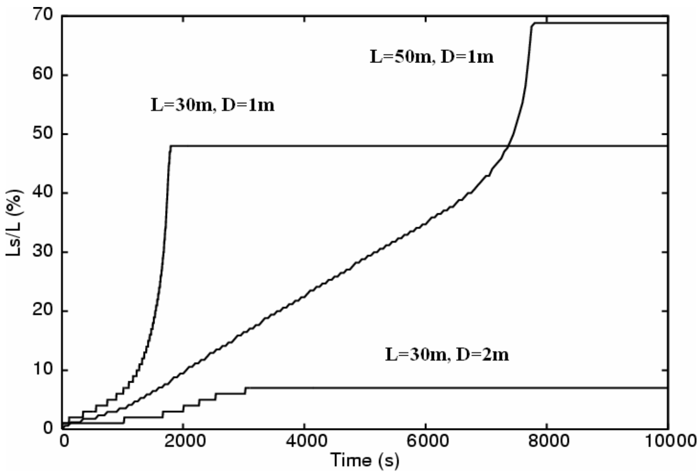


Fig. 5. Evolution with time of the normalized saturated length for three geometries

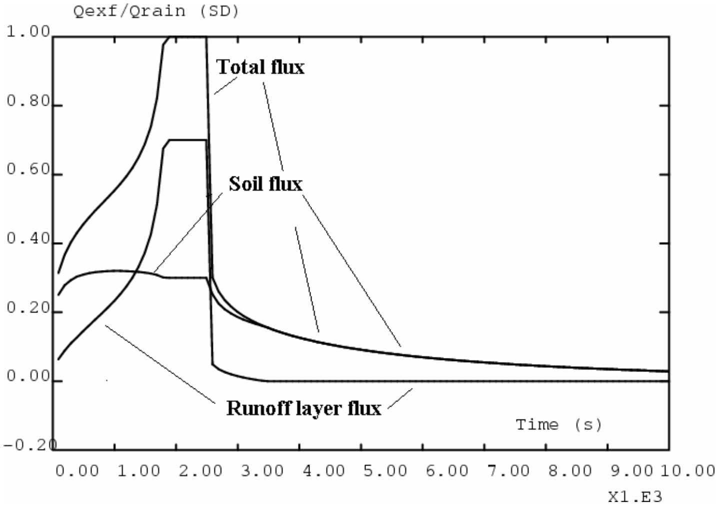


Fig. 6. Evolution with time of normalized outgoing water fluxes

CONCLUSION

The objective of this work is to develop a modelling approach which allows to model the water cycle and the interactions between surface and subsurface processes in a continuous way. We use the diffusive wave approximation to model runoff, and treat this equation as a nonlinear Darcy one. We introduce a runoff layer at the surface in which surface processes are simulated. A single nonlinear Darcy equation with domain-dependent parameters, describing all the S/SS processes, is obtained. This model has been validated with many test cases and this paper presents some of them. From now we plan to study the impact of rainfall annual variability on the water budget in a catchment, and at a longer term the impact of climate change on the hydrological response of a given watershed. In particular we will study, for given climate or meteorologic conditions, the respective role of surface and subsurface compartment in the hydrological response of a watershed.

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